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SRM Propellant and Polymer Materials Structural Modeling

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Carleton J. Moore
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

NASA
National Aeronautics
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The author would like to thank Mr. Verderaime of NASA MSFC for the review and discussions which helped play a part in the documented investigation and the resulting papers. Also, the author would like to thank Dr. Chamis of NASA LeRC for his discussions and his moral support of development of a new model using a power equation which deviates from classic viscoelastic math models.

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INTRODUCTION

The need for better structural characterization and analysis of the solid propellant grain is dictated by the need to characterize the statics and dynamics of the solid rocket boosters as a system. There, presently, is not a data base of analysis methods and supporting test data which allows an accurate prediction of the propellant or bondline strain or stress. This lack of test and analytical structural characterization of propellants results in unknown safety factors and a general inability to develop criteria for the quality control necessary for manned space systems. In fact, the propellant cumulative structural margins for storage, thermal cycling, transportation induced stress cycling, and ignition transients have never been experimentally verified on any manned solid rocket motor (SRM).

The SRM propellant, insulation, inhibitor, liners, and seals have been generally characterized as being made of viscoelastic materials. Although the viscoelastic classification has been generally accepted, close examination of these materials reveal that they are either more complex and nonlinear than classic viscoelastic models or the actual mechanisms should be redefined in a different mathematical form. The following investigation reviews and evaluates the Space Shuttle SRM propellant structural analyses utilizing stress relaxation modulus data. The stress relaxation test data is examined and a new math model is proposed.

STRESS RELAXATION MODULUS

The Stress Relaxation Modulus is obtained by inducing a constant strain or shear in the propellant test specimen and measuring the reaction boundary forces or moments with respect to time. The stress relaxation modulus at 70°F is tabulated versus time in the two left hand columns of Table 1. The data of Table 1 was obtained from Reference 4. The stress relaxation modulus data points, for time greater than 10^{-8} min, are plotted in Figure 1. Table 2 and Figure 2 show the temperature dependent time shift factor which is used by Reference 4 to extrapolate the test results of Figure 1 to different temperatures. Time is simply divided by this factor which has the effect of expanding or compressing the material time scale with respect to the actual time scale of the transient events. The modification of the Stress Relaxation Modulus by the time shift parameter is consistent with the assumption fo the material being linearly viscoelastic. Although the temperature dependent time shift parameter is generally accepted, the author's opinion is that the nonlinear boundaries of the real material must be defined to realistically judge the blanket usage of this assumption. There is no doubt a theoretical region where the temperature dependent time shift parameter is suitable, but as the polymer is applied theoretically and extended to nonlinear cases other than stress relaxation, this assumption must be evaluated closely.

TABLE 1. PRONY SERIES TERMS FOR SHEAR MODULUS

t (min)	G(t) psi	G_i (psi)	Log λ_i
		$G_0 = 75$	
10^6	80	$G_1 = 8.24$	Log $\lambda_1 = 6.3$
10^4	92	$G_2 = 14.51$	4.3
10^2	130	$G_3 = 29.4$	2.3
1	220	$G_4 = 153.4$	0.3
10^{-2}	420	$G_5 = 231.3$	-1.7
10^{-4}	1,100	$G_6 = 1,013.0$	-3.7
10^{-6}	2,930	$G_7 = 2,367.0$	-5.7
10^{-8}	10,000	$G_8 = 11,023.$	-7.7
10^{-10}	48,500	$G_9 = 55,508$	-9.7
10^{-12}	220,000	$G_{10} = 247,120$	-11.7

TABLE 2. Log a_T VERSUS TEMPERATURE

Log a_T	Temperature, °F
-4.4	145
-2.0	105
0	70
2.1	33
4.4	0
6.7	-30

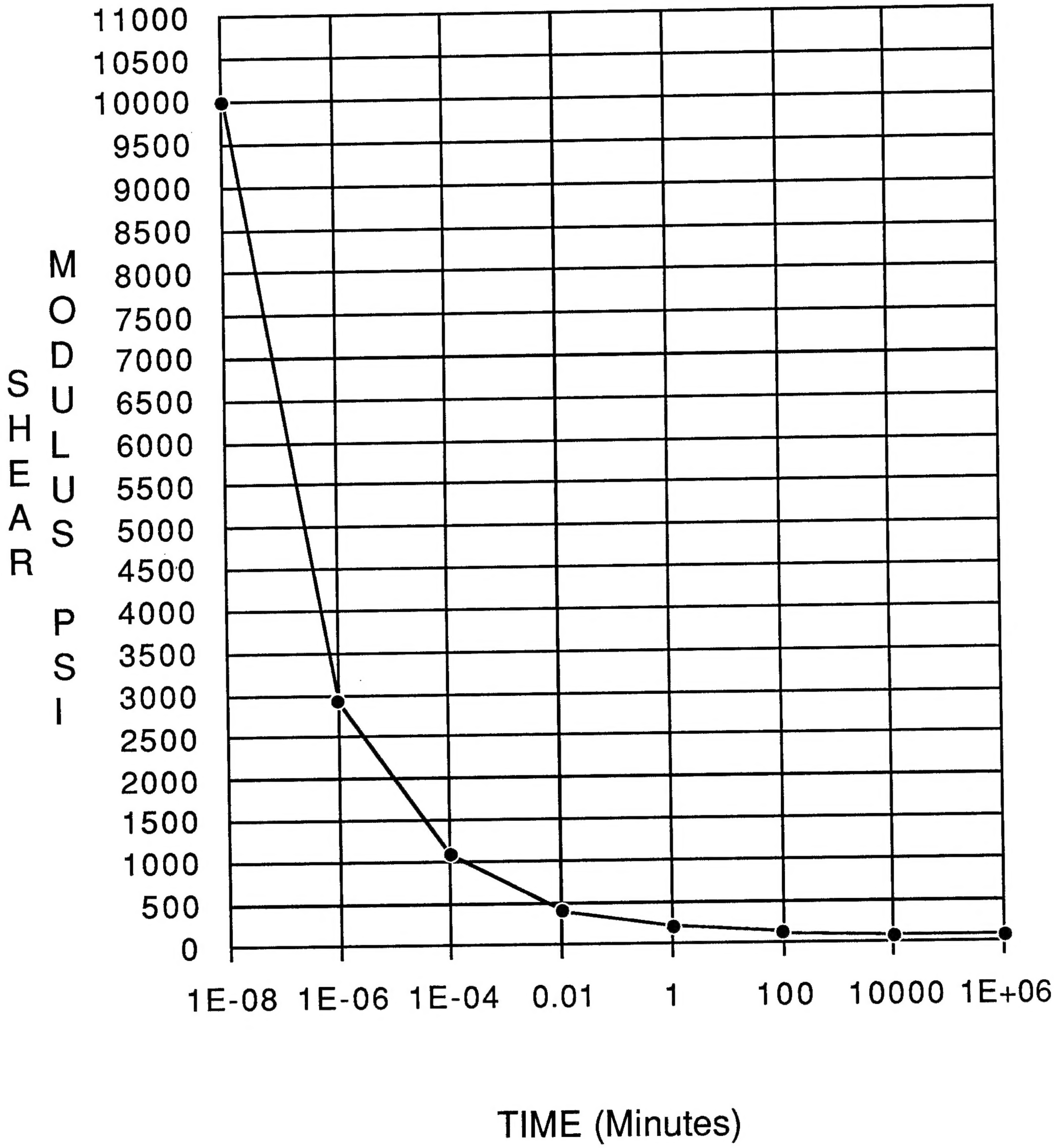

Figure 1. Shear modulus G(t) versus log time.

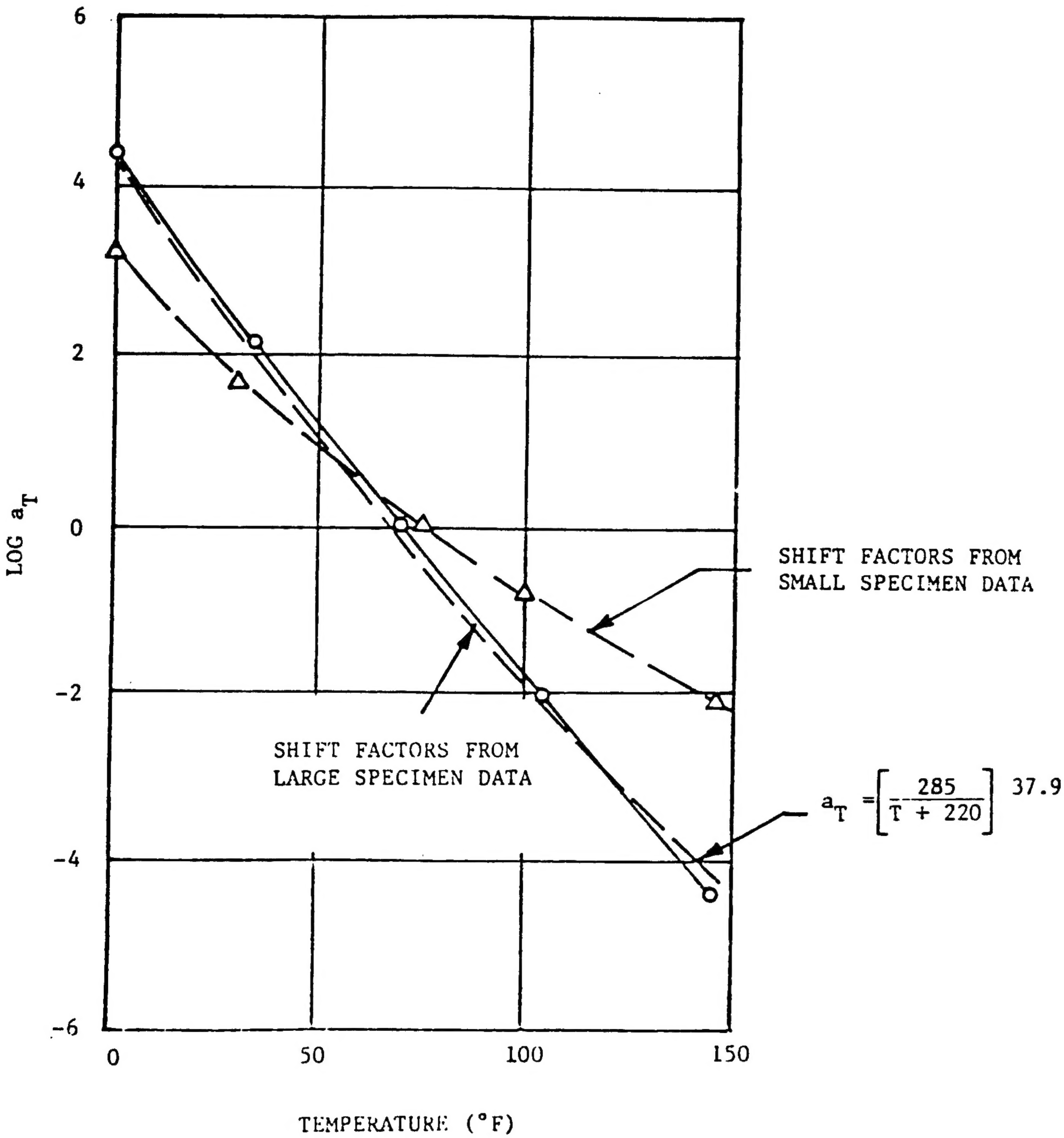

Figure 2. Log a_T versus temperature, large and small specimen stress relaxation data.

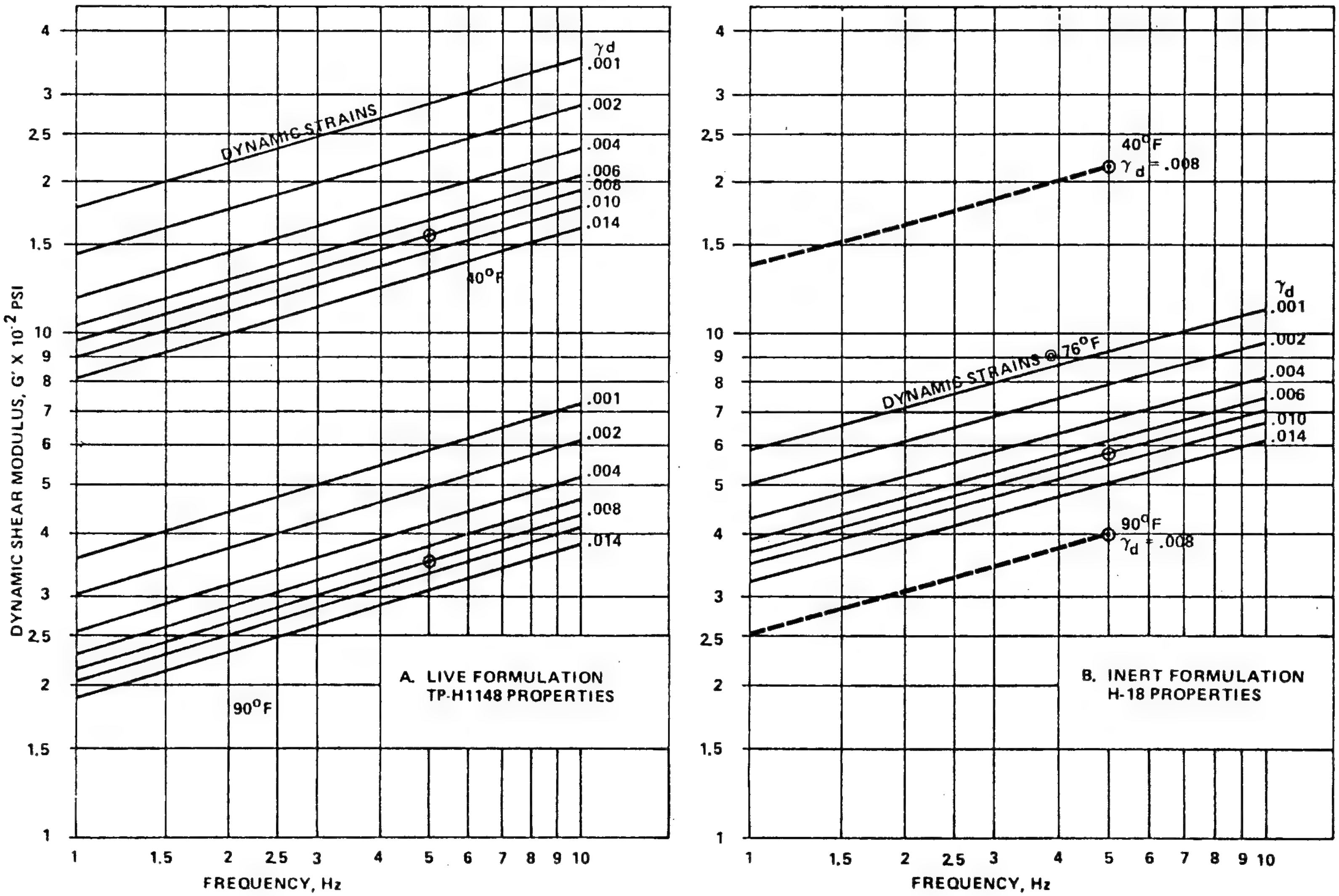

Figure 3. SRB propellant real modulus versus frequency and dynamic strains.

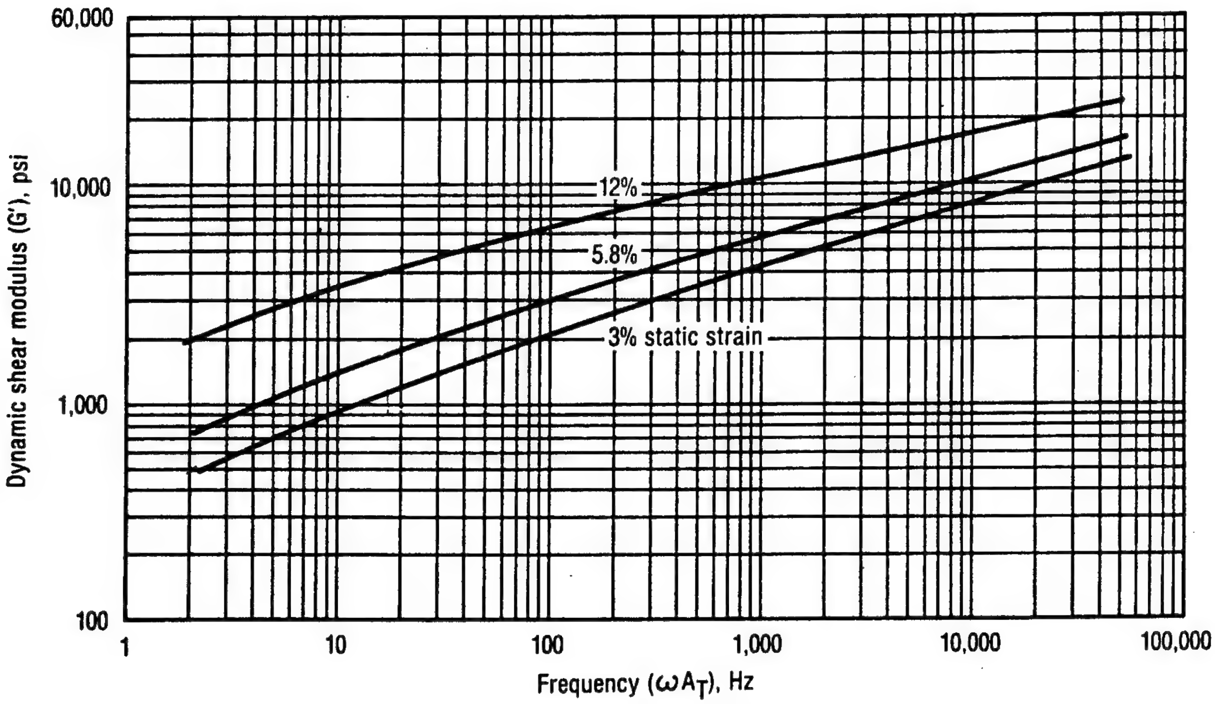

Figure 4. Comparison of static strain levels for batch No. TP-H1148-9.

The stress relaxation data has been used by Reference 4 for propellant structural analysis of the Space Shuttle ARM. One method is simply for the engineer to assume or select a time during an event such as the long term storage, then the modulus is defined by the stress relaxation data and a linear elastic analysis is completed for that one instant in time. It should be remembered that the stress relaxation modulus is only valid for constant strain while many actual problems are really transient in nature. It has been proven in test that under transient conditions the modulus is strain rate and static strain dependent (Figs. 3 and 4).

The second method of analysis is to use a finite element linear viscoelastic transient computer code. The code currently used by Reference 4 uses a Prony series to model the modulus of the assumed linear viscoelastic material, as shown below (Table 1):

$$G(t) = G_0 + G_1 e^{-t/\lambda_1} + G_2 e^{-t/\lambda_2} + G_3 e^{-t/\lambda_3} + \dots$$

Reference 4 used only the stress relaxation data to define the terms in this series, and thus, again the model as currently used may only be valid for the constant strain condition, which is hardly a typical transient condition.

The Prony series material model (Fig. 5) can be defined conceptually by a very simple schematic consisting of a spring in parallel with a number of Maxwell elements, as in Reference 1. The Maxwell elements are simply a spring in series with a viscous dash pod damper. This is the simplest and most numerically convenient viscoelastic model for a material. A time dependent analysis is then done with this model assuming linear superposition and ignoring a variety of factors which can cause the effective modulus to shift by factors of two and more.

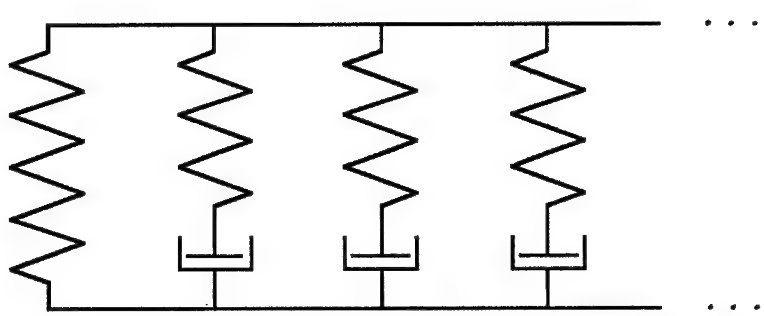

Figure 5. Conceptual schematic of Prony Series material model.

INHERENT PRONY SERIES SRM MATH MODEL WEAKNESSES

The Space Shuttle SRM Prony Series structural model has several inherent weaknesses numerically, experimentally, and theoretically.

1. A large number of terms are used by Reference 4 to match a comparable number of stress relaxation test points. (11 terms with a total of 21 constants as shown in Table 1.)

2. The series is extremely ill conditioned. There are many combinations of G_1, G_2, G_3, ..., and λ_1, λ_2, λ_3, ..., that can be fit to data generated by exact functions to four significant figures. The introduction of experimental noise makes the operation hopeless [2].

3. The series, as fit in Reference 4, does not have a smooth uniform curve between the stress relaxation data points even though it passes through the data points (Fig. 6).

4. The derivation of the dynamic modulus from the Prony Series using the assumption of linear superposition does not match experimental data in magnitude or trend (Figs. 7 and 8). The real part of the complex modulus is labeled as G'. The imaginary part of the complex modulus is labeled as G''. The magnitude of the complex modulus is labeled as G^*.

5. Current linear viscoelastic codes and Prony Series models do not account for the effects of static strain and dynamic strain (strain rate) on dynamic modulus (Figs. 3 and 4).

6. The original Space Shuttle SRM propellant model was based on test data only valid for constant strain (Reference 4).

7. The series fit to constant strain test data does not have in itself any relation to the material response unless λ_1, λ_2, λ_3, ... are selected properly based on understanding of the response phenomena and the material mechanisms.

8. The region of the lowest propellant safety factor is the forwrd segment star pattern. Although the constant strain test assumption may be very good for the round smooth bore segments under constant pressure, the assumption is not valid for the star pattern region.

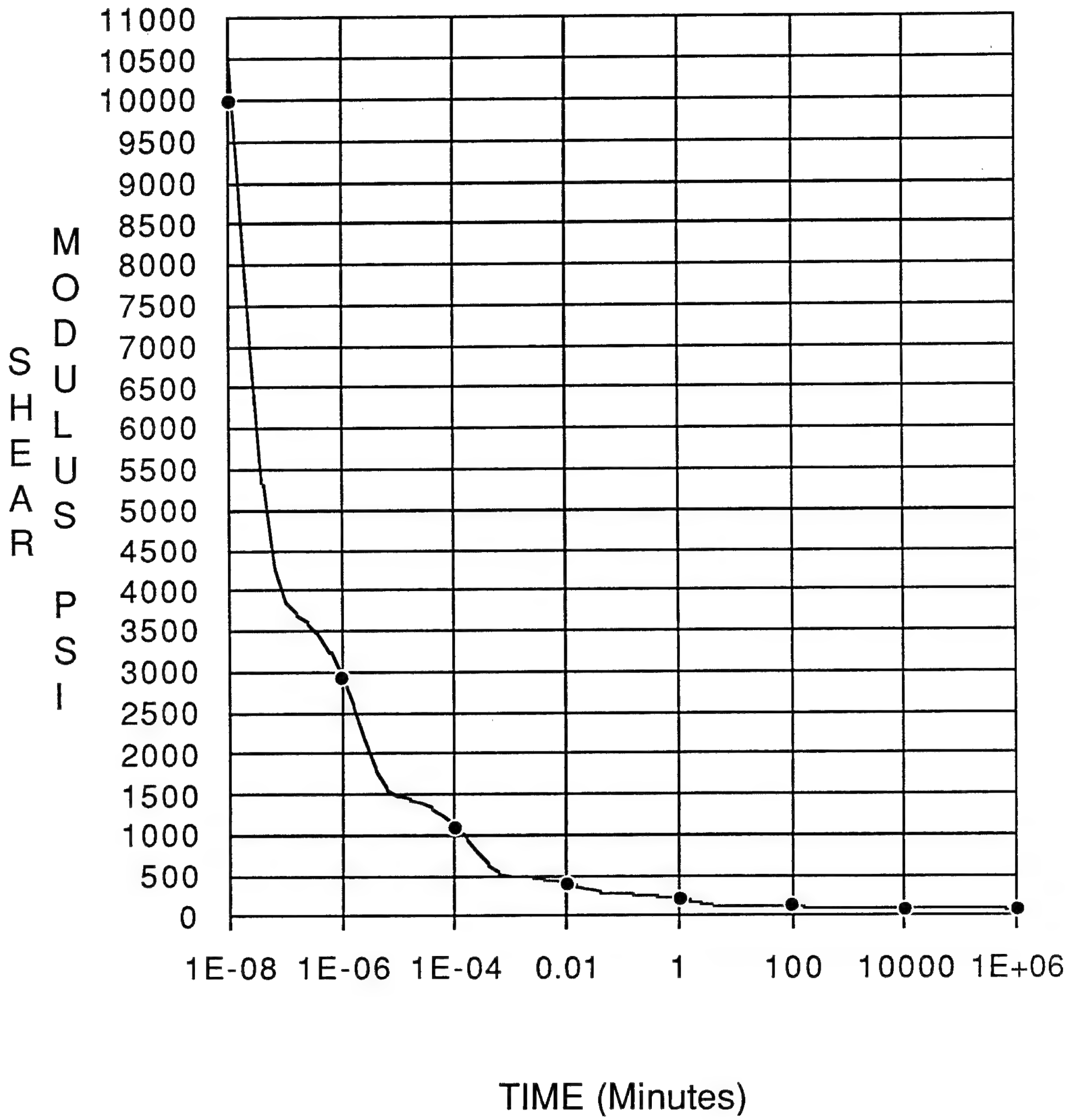

Figure 6. Shear modulus G(t) versus log time, Prony Series fit.

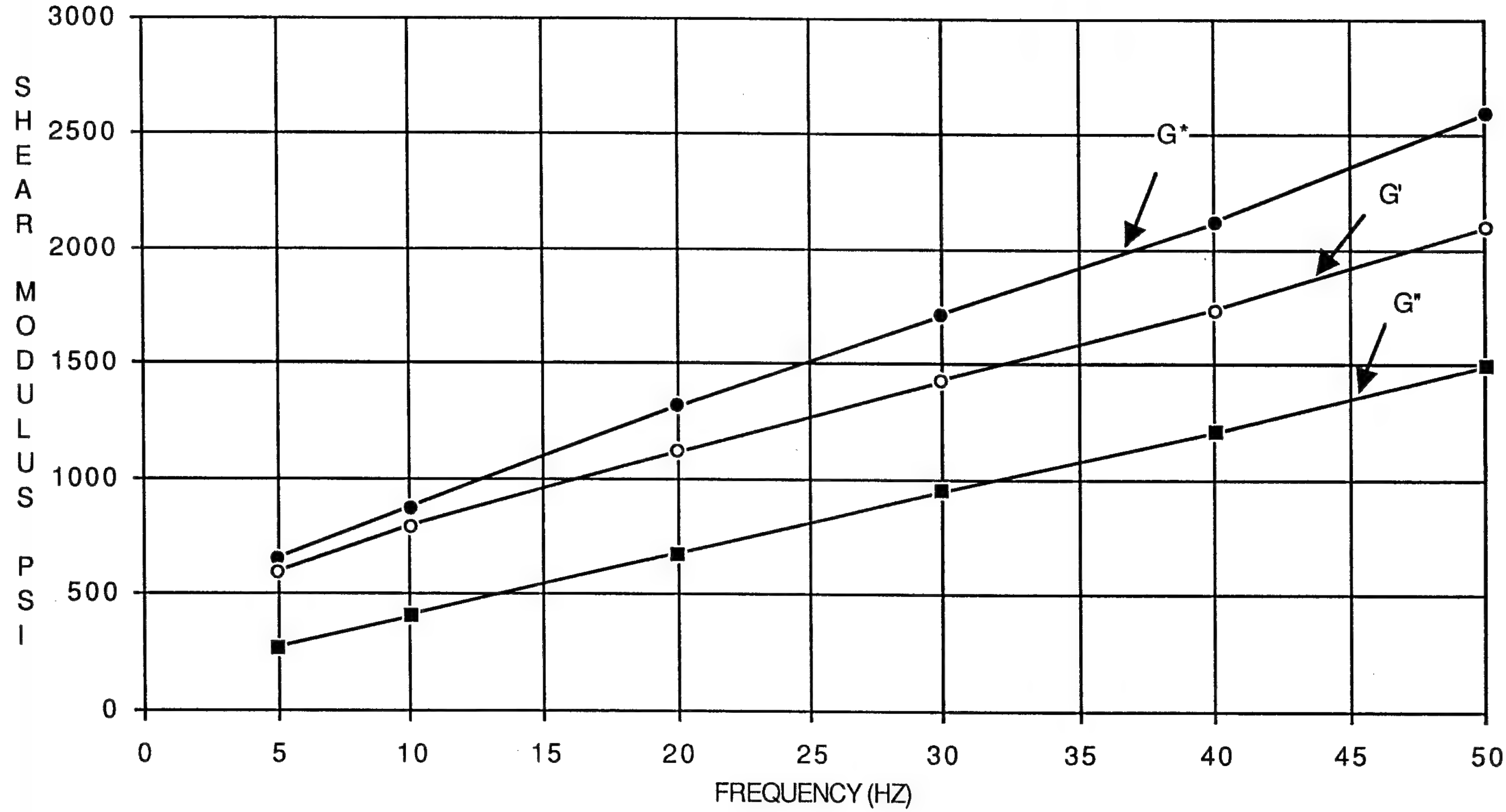

Figure 7. Dynamic shear modulus of TP-H1148 SRM propellant, Mix 9910060 at 70°F, Sample 36 from TWR-11779.

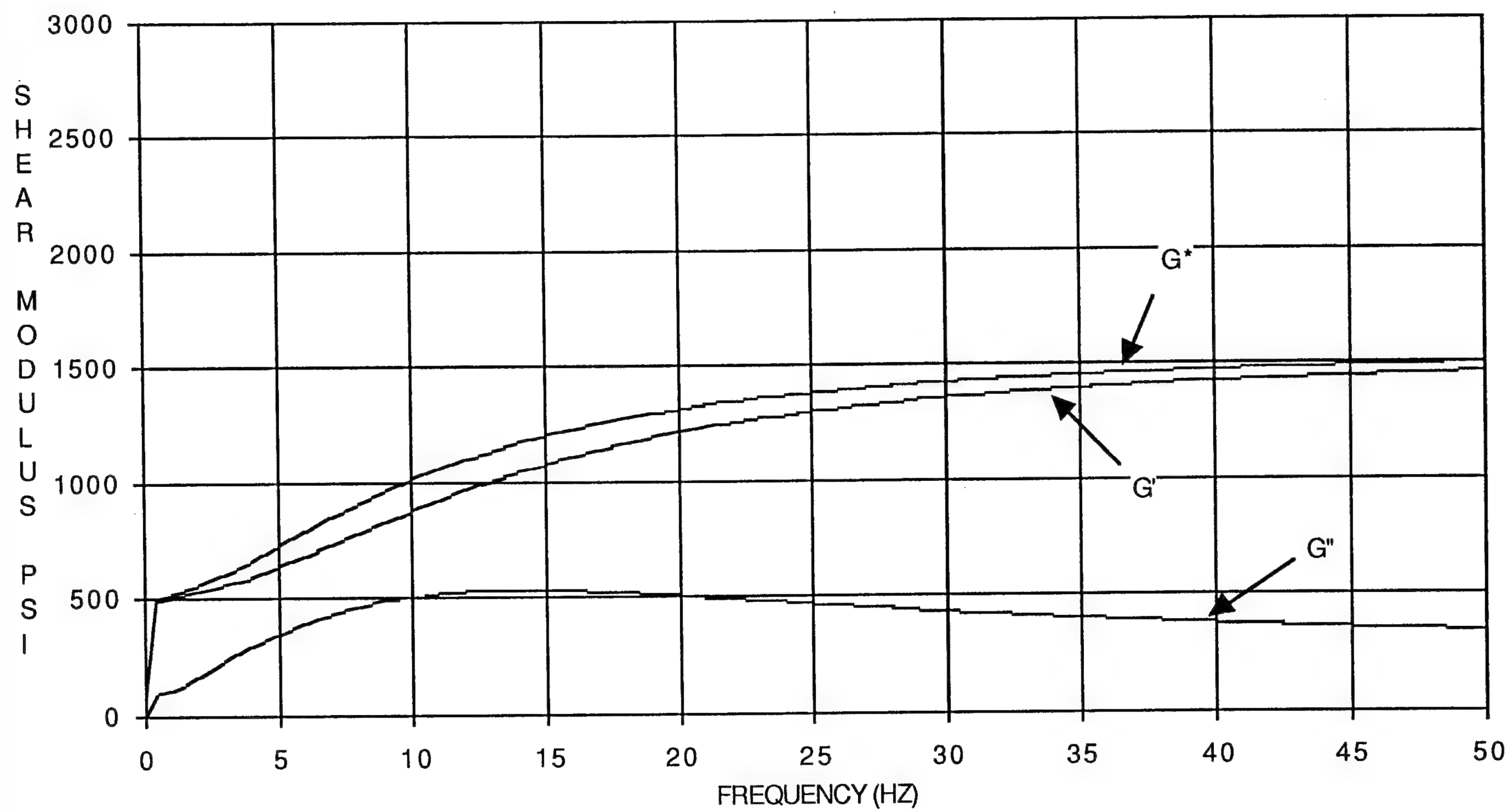

Figure 8. Dynamic shear modulus of TP-H1148 SRM propellant based on Prony Series fit of stress relaxation data.

SRM REDESIGN ANALYSIS TECHNIQUES

The Space Shuttle SRM redesign was checked by an independent and direct use of propellant test data. The redesign structural analysts have modeled the propellants and insulation as elastic isotropic materials with an analysis provision or capability of large strains and a variational stress relation which allows more accurate analyses of nearly incompressible materials (References 10 and 12). The nonlinear variation in the material properties was accounted for by reviewing test data and selecting isotropic properties which were the best approximation for the particular load case. Proper review of the validity of these analyses requires a presentation of the utilized test data and a review of the selection of the equivalent elastic isotropic material data (Reference 10). There was no presentation of a test verification of the material property selection and the resulting finite element math model.

The analysis of the operational load cases involving a combination of load cases of a different nature, such as pressure, thermal, and acceleration, are analyzed separately with totally different material properties selected for each case. These separate cases are then linearly added for the operational condition. The linear superposition with the limiting constraints and assumptions of the above analysis is consistent for small strains. It is not mathematically correct to apply linear superposition to a large strain analysis. There has been no test verification of the actual superposition relations of these nonlinear materials.

The dynamic transportation analysis was termed very crude by the author of the reviewed report (Reference 10) and in his own words should be considered only an approximate with a more detailed analysis recommended. This analysis first calculated a frequency of the region in question by iterating with assumed shear modulus and dynamic modulus test data from Reference 8. It should be cautioned that due to the sensitivity of the dynamic modulus to static strain and strain rate this test data is probably only valid for the test conditions and as documented can not be accurately extended within a factor of 4 to the SRM. After a frequency was found, a g level or environmental specification was obtained from Reference 11. A load and resulting strains were calculated using a linear elastic static solution with factors applied for transmissibility at resonance with a 33 percent damping. Although the dynamic modulus test data can be questioned, a state of the art solution using this test data would have been a frequency response analysis using the transportation environment as driving forces.

The above analysis reflects the need for the development of a test verifiable math model for the SRM propellant and related polymer materials. The above reviewed work needs a demonstrated and documented test verification. Also, the analyses is totally dependent on the engineer's selection of material data with no test verified and NASA endorsed standards.

NEW STRESS RELAXATION MODEL

The weaknesses of the fit experimental data to the Prony Series prompted an investigation of the test data as shown in Table 1. The shear modulus versus time of Figure 1 is replotted in Figure 9 on the log log scale. It is seen that the table data points can be fit with a single parabolic curve. Both Figures 1 and 9 show that the shear modulus is approaching a constant static value. A further simplification can be made by subtracting out the static modulus and replotting the dynamic shear modulus versus time on a log log scale, as shown in Figure 10. As plotted in Figure 10, the stress relaxation data looks like it can be fit with straight lines. A straight line would do a good job of fitting data taken at times greater than 10^{-8} min. The data points for less than 10^{-6} min either indicate a different phenomena or more likely problems with testing or data reduction techniques employed in Figure 1 and Table 1 from Reference 4. The stress relaxation modulus can be related by the following simple equation.

$$G(t) = G_0 + G_1 t^{-n}$$

where

t = time in seconds

$G_0 = 75$ psi

$G_1 = 331$ psi

$n = 0.2255$.

Figure 11 is a plot of the above equation and should be compared to Figure 6. The above equation can still be used with the temperature dependent time shift parameter.

The theoretical dynamic shear modulus can be derived from the above power equation, assuming linear superposition as follows.

Real Modulus

$$G'(\omega) = \omega \int_0^\infty \sin(\omega s)\, G(s)\, ds \quad \text{[Ref. 1]}$$

$$G'(\omega) = G_0 + G_1\, \omega \int_0^\infty [\sin(\omega s)/s^n]\, ds$$

$$G'(\omega) = G_0 + G_1\, \omega^n\, \Gamma(1-n) \cos(n\pi/2)$$

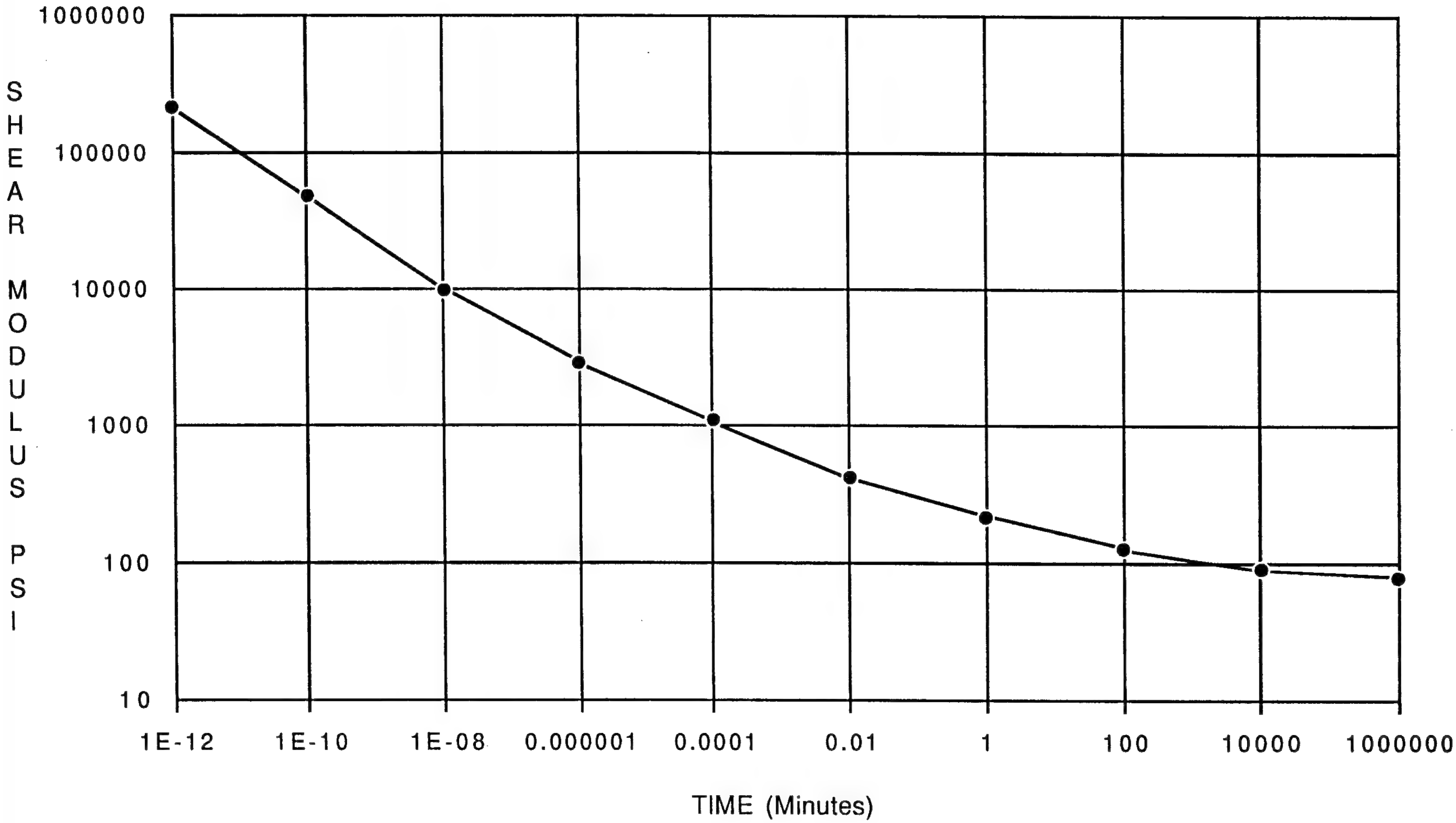

Figure 9. Log shear modulus G(t) versus log time.

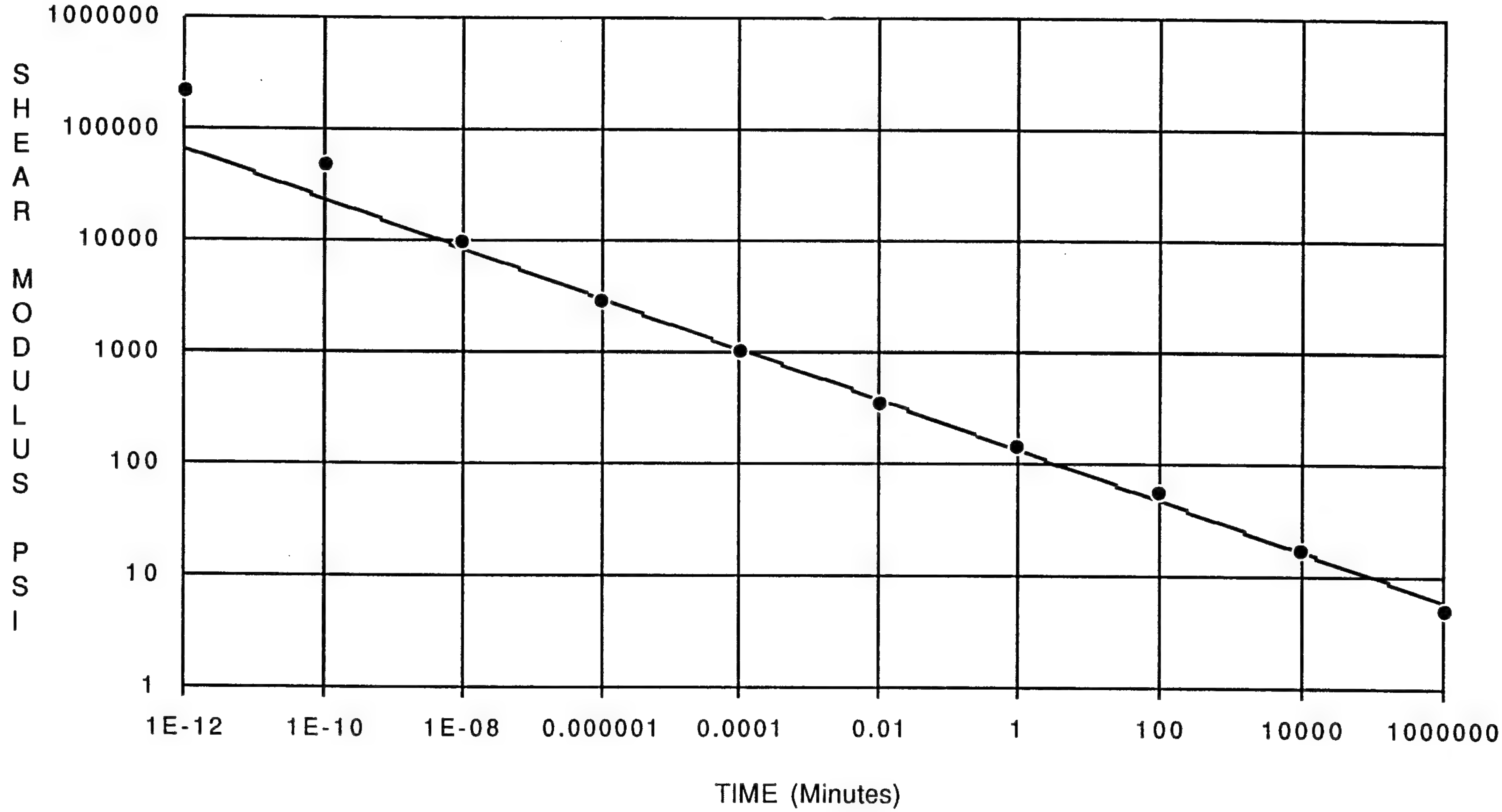

Figure 10. Log shear modulus (G(t)-75) versus log time, power equation fit.

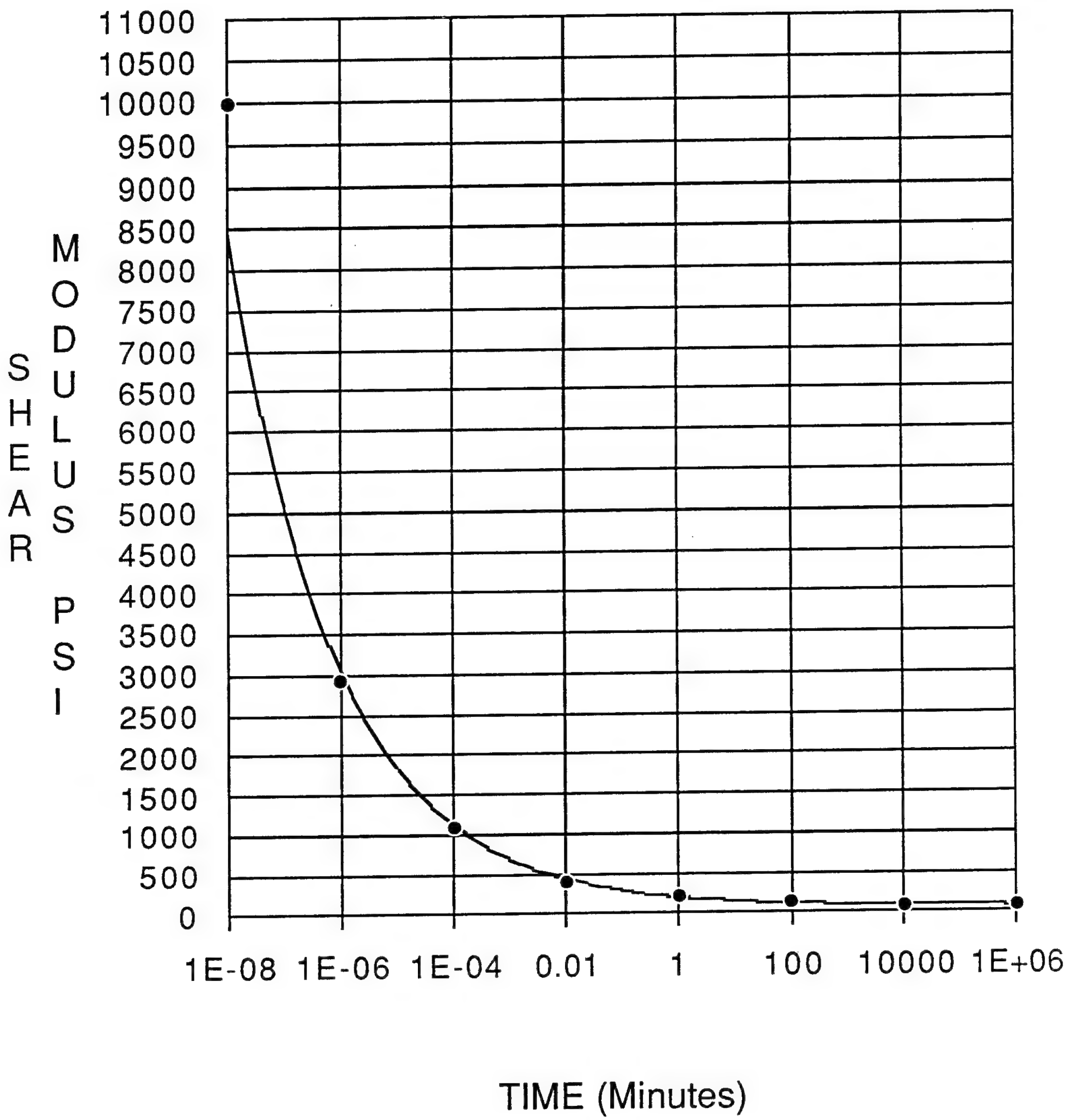

Figure 11. Shear modulus G(t) versus log time, power equation fit.

Imaginary Modulus

$$G''(\omega) = \omega \int_0^\infty \cos(\omega s)\, G(s)\, ds \quad \text{[Ref. 1]}$$

$$G''(\omega) = G_1\, \omega \int_0^\infty [\cos(\omega s)/s^n)\, ds$$

$$G''(\omega) = G_1\, \omega^n\, \Gamma(1-n)\, \sin(n\pi/2)$$

Complex Modulus

$$G^*(\omega) = \sqrt{G'(\omega)^2 + G''(\omega)^2}$$

The above result is plotted on Figure 12.

The above derived dynamic modulus can be compared to the theoretically defined dynamic modulus based on the Prony Series [1].

Prony Real Modulus

$$G'(\omega) = G_o + \sum_{i=1}^{N} \frac{G_i\, \omega^2\, \lambda_i^2}{1+\omega^2\, \lambda_i^2}$$

Prony Imaginary Modulus

$$G''(\omega) = \sum_{i=1}^{N} \frac{G_i\, \omega\, \lambda_i}{1+\omega^2\, \lambda_i^2}$$

Prony Complex Modulus

$$G^*(\omega) = \sqrt{G'(\omega)^2 + G''(\omega)^2}$$

The above result was plotted on Figure 8.

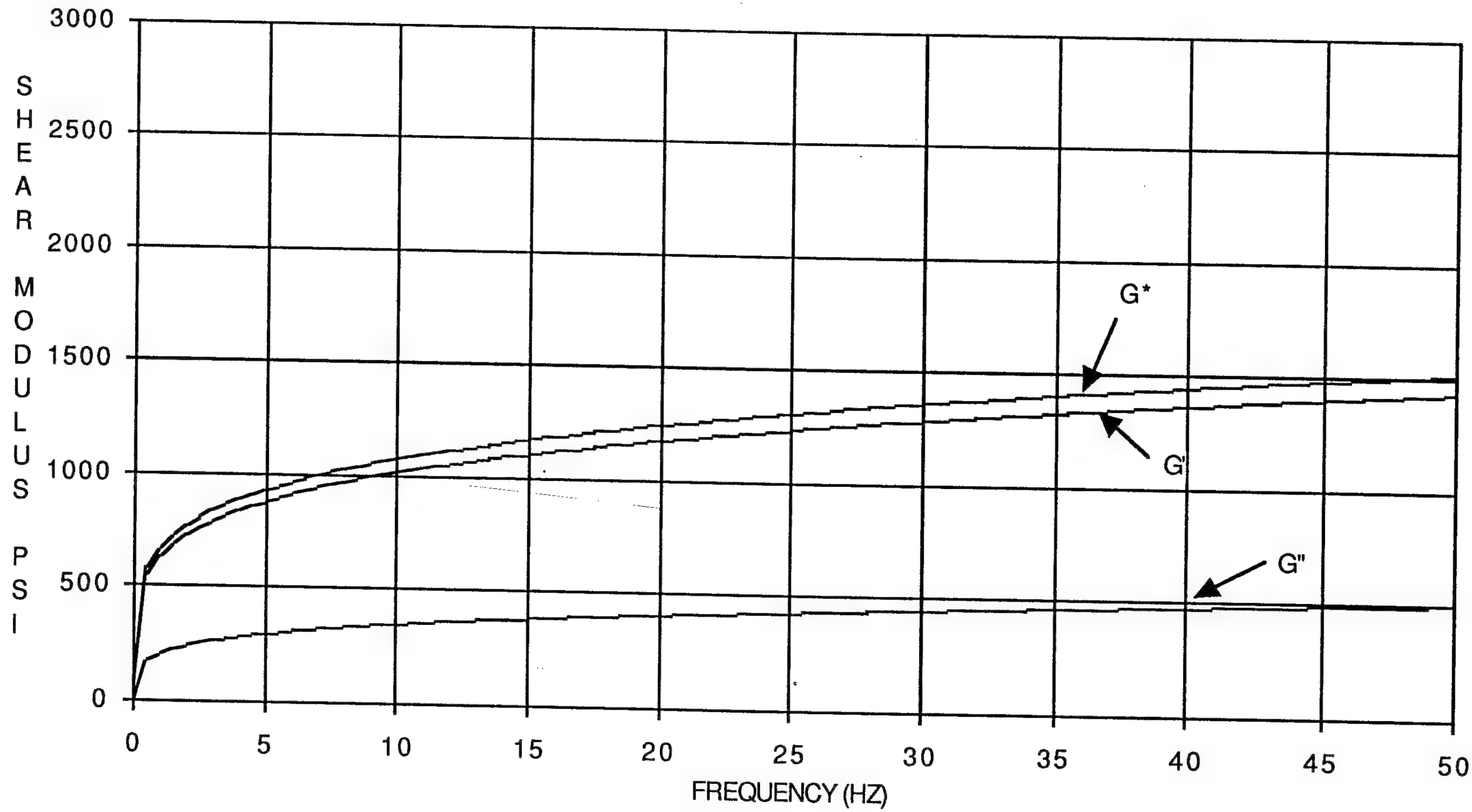

Figure 12. Dynamic shear modulus of TP-H1148 SRM propellant based on power equation fit of stress relaxation data.

Although the power equation theoretically derived dynamic modulus (Fig. 12) was based on the same stress relaxation data, the results are very different from the result shown in Figure 8. The Figure 8 results were derived from the Prony Series. It can be seen that the derived dynamic shear modulus plotted in Figures 8 and 12 do not compare to the experimental results of Figure 7 very well. The experimental test data obtained from Reference 8 as shown in Figure 7 was documented as being driven with an electromechanical vibrator at a constant displacement. If the experimental results of Figure 7 were actually the result of being driven at a constant maximum displacement then as the frequency increased the velocity and strain rate increased in proportion. Figures 3 and 4 show that the dynamic shear modulus is significantly shifted by changes in the dynamic and static strains. If the experimental data had been driven at a constant maximum velocity and strain rate, the experimental results probably would have been different.

The theoretically derived dynamic shear modulus based on either the Prony Series or the Power Equation, as they currently stand cannot predict this shift with dynamic and static strain. These effects must be included and any testing or analysis done without the proper attention to these parameters is of questionable nature. If the stress relaxation modulus is to be related to creep tests, dynamic modulus tests, and strain rate tests, these two parameters must be included and checked for temperature dependency and validity of the current time shift parameter.

The author would like to propose a new model for the material properties of the propellant and other polymer materials. The modulus can be defined by a summation of terms where each term is a function of multipliers. Each multiplier is of two possible forms. The first is a power equation and the second is an exponential of a power equation.

An example of the series can be

$$G = G_{Static} + G_{Dynamic}$$

where, G_{Static} may reduce to a constant such as 75 psi for the propellant and $G_{Dynamic}$ may be defined as a function of time, temperature, static strain, dynamic strain, and other terms as needed.

$$G_{Dynamic}(t,T,\varepsilon,\dot{\varepsilon}) = M(t)\,M(T)\,M(\varepsilon)\,M(\dot{\varepsilon})$$

where M(X) takes two possible forms:

$$M(X) = C_1\,\beta_M(X)$$

or

$$M(X) = C_1\,e^{C_2\beta_m(X)}$$

where

$$\beta_m(X) = \left[\frac{\sum_{i=0}^{m} C_{N_i} (X - X_R)^i}{\sum_{i=0}^{m} C_{D_i} (X_0 - X_R)^i} \right]^{P_x}$$

X_0 may be equal to X or an original value of X

X_R is a X reference value

C_{N_i} is a constant in the numerator

C_{D_i} is a constant in the denominator

P_X is the power of the function of X .

As an example, the first propellant model would probably use a β function of order one.

$$\beta_1(X) = \left[\frac{C_{N_0} + C_{N_1} (X - X_R)}{C_{D_0} + C_{D_1} (X_0 - X_R)} \right]^{P_x}$$

The first choice for the propellant multiplier would be:

$$M(X) = C_1 \, \beta_1(X)$$

Applying the model to the current test results for stress relaxation modulus

$$M(t) = 331 \, t^{-0.225}$$

$$M(T) = \left[\frac{285}{T+220} \right]^{8.5275} \qquad \text{(adapted from Fig. 2)}$$

$M(\varepsilon)$ not yet defined

$M(\dot{\varepsilon})$ not yet defined

The exponential form of the multiplier could model the Prony Series form and also adapt to past accepted models of different polymers. As an example, the temperature dependent time shift parameter that has been accepted for polyisobutylene, natural rubber, polyurethane elastomer, polystyrene, and poly(ethyl methacrylate) is as follows from Reference 1.

$$\log \alpha_T = \frac{-C_3(T - T_R)}{C_4 + T - T_R}$$

The temperature dependent multiplier would be of the following form for this relation.

$$M(T) = C_1 e^{C_2 \left(\frac{-C_3(T - T_R)}{C_4 + T - T_R} \right)^{P_T}}$$

CONCLUSIONS

A new math model for stress relaxation has been proposed. This model has been extended and generalized into a form utilizing multipliers with each multiplier incorporating the contribution of one important parameter such as time, temperature, strain rate, static strain, etc. The final definition of all the necessary terms and the general application of this model must be developed and verified by detailed consistent test program for the materials in question as in Reference 9. Although, it is hoped this model will be very successful, it is merely the next step which is highly dependent on practical test results, and it must be suitably test verified.

APPENDIX

Additional plots used in the evaluation of the equation forms of the stress relaxation modulus and the dynamic modulus are given in Figures 13 through 23.

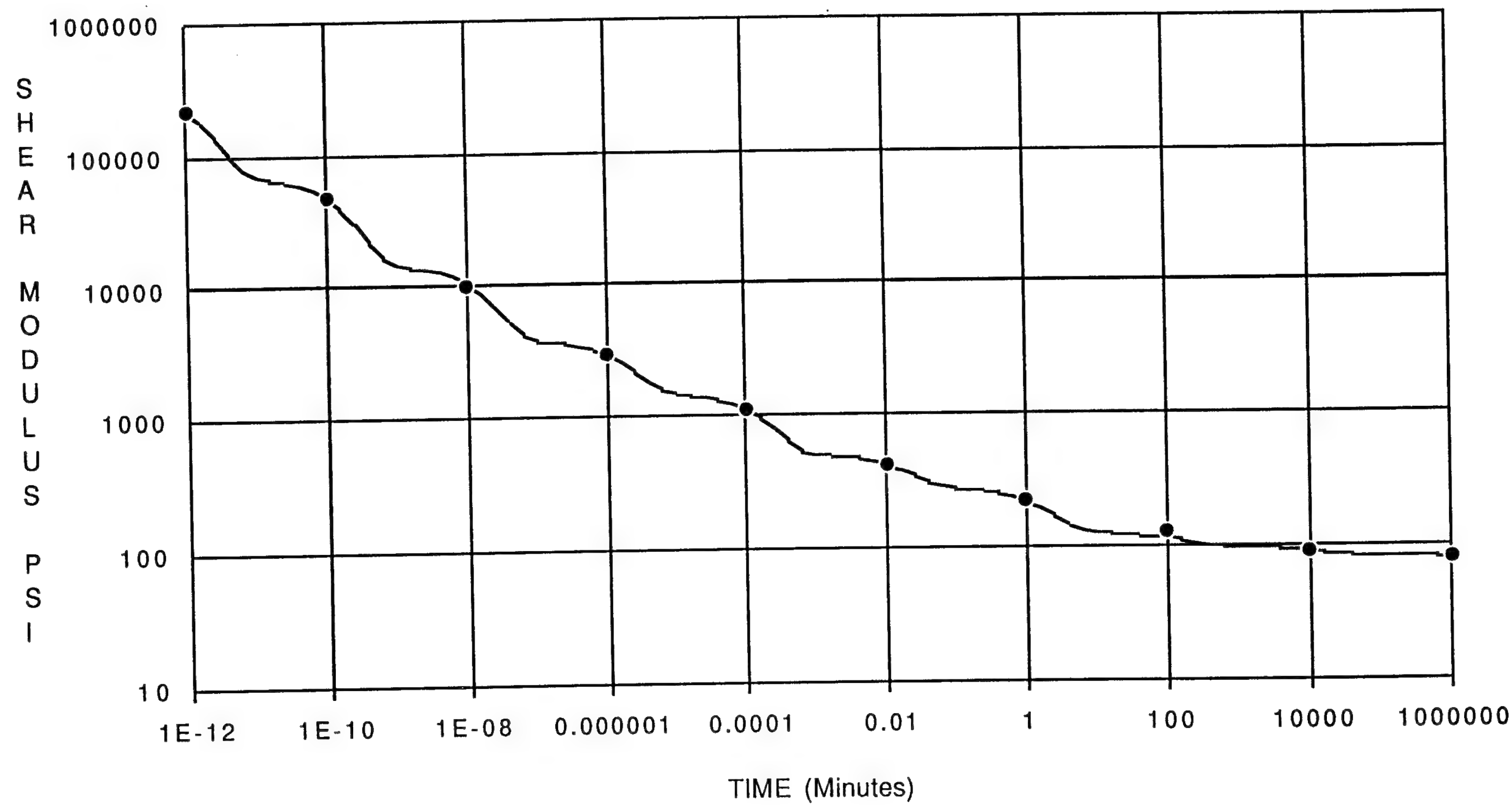

Figure 13. Log shear modulus G(t) versus log time, Prony Series fit.

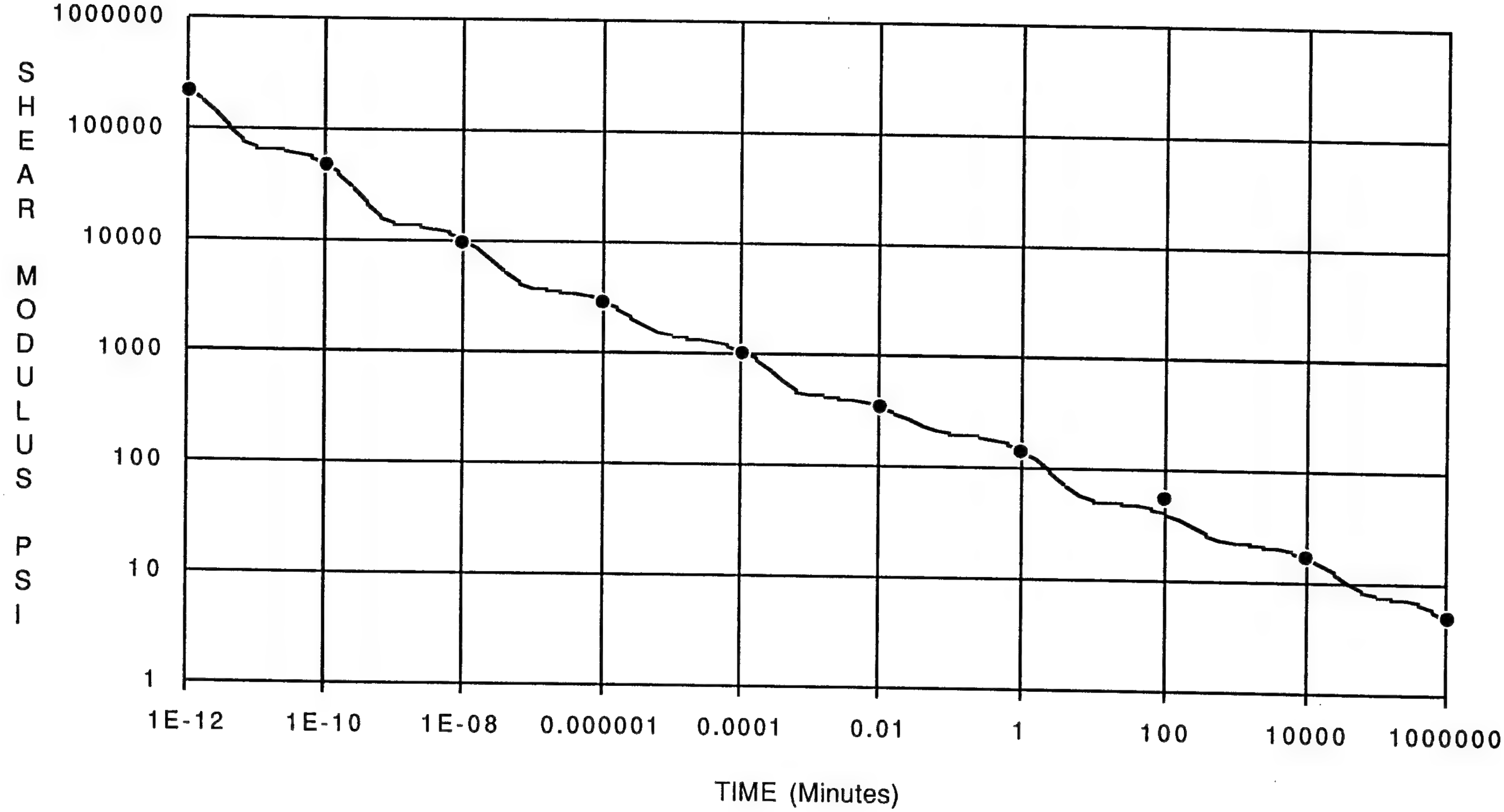

Figure 14. Log shear modulus (G(t)-75) versus log time, Prony Series fit.

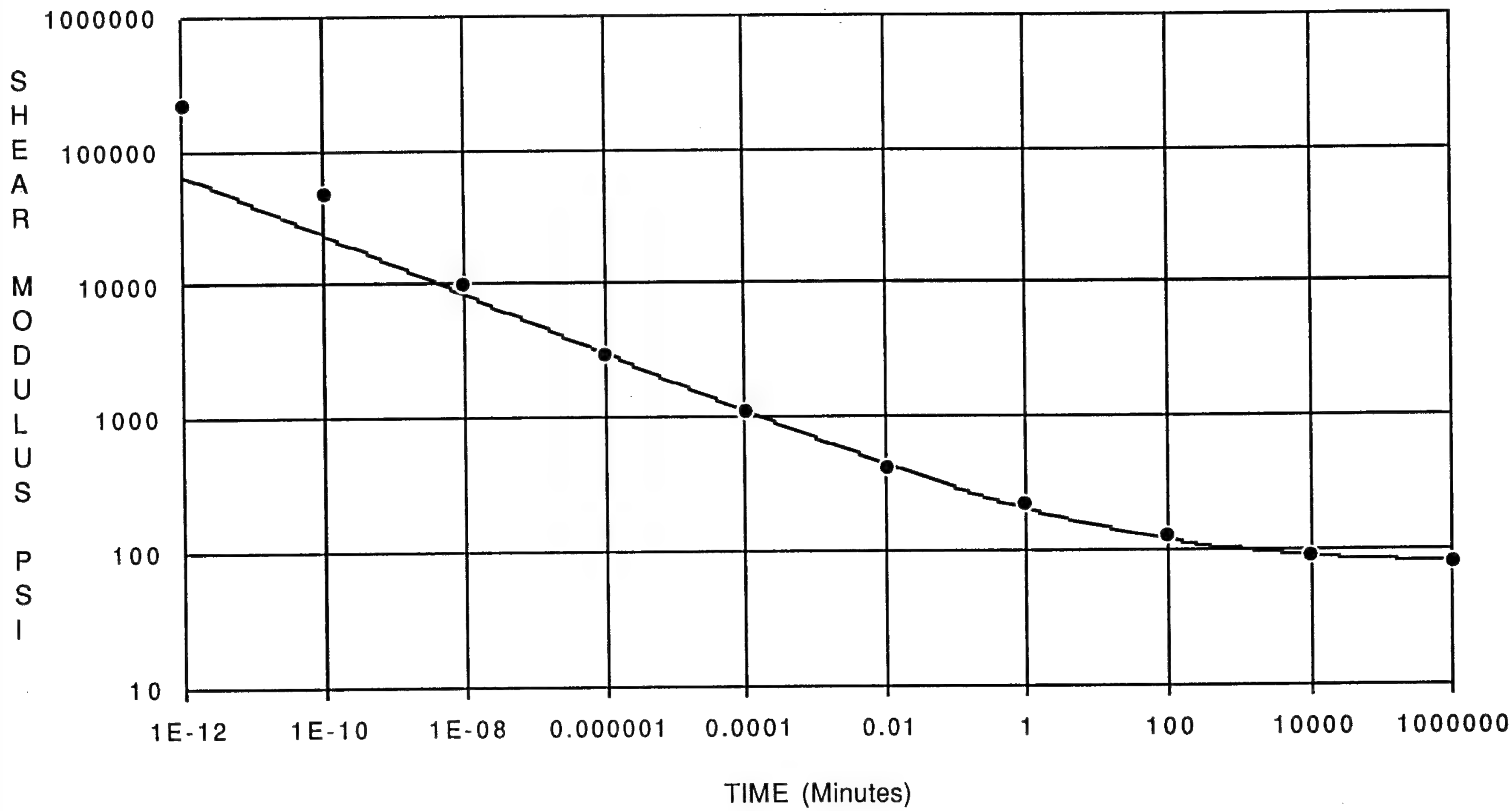

Figure 15. Log shear modulus G(t) versus log time, power equation fit.

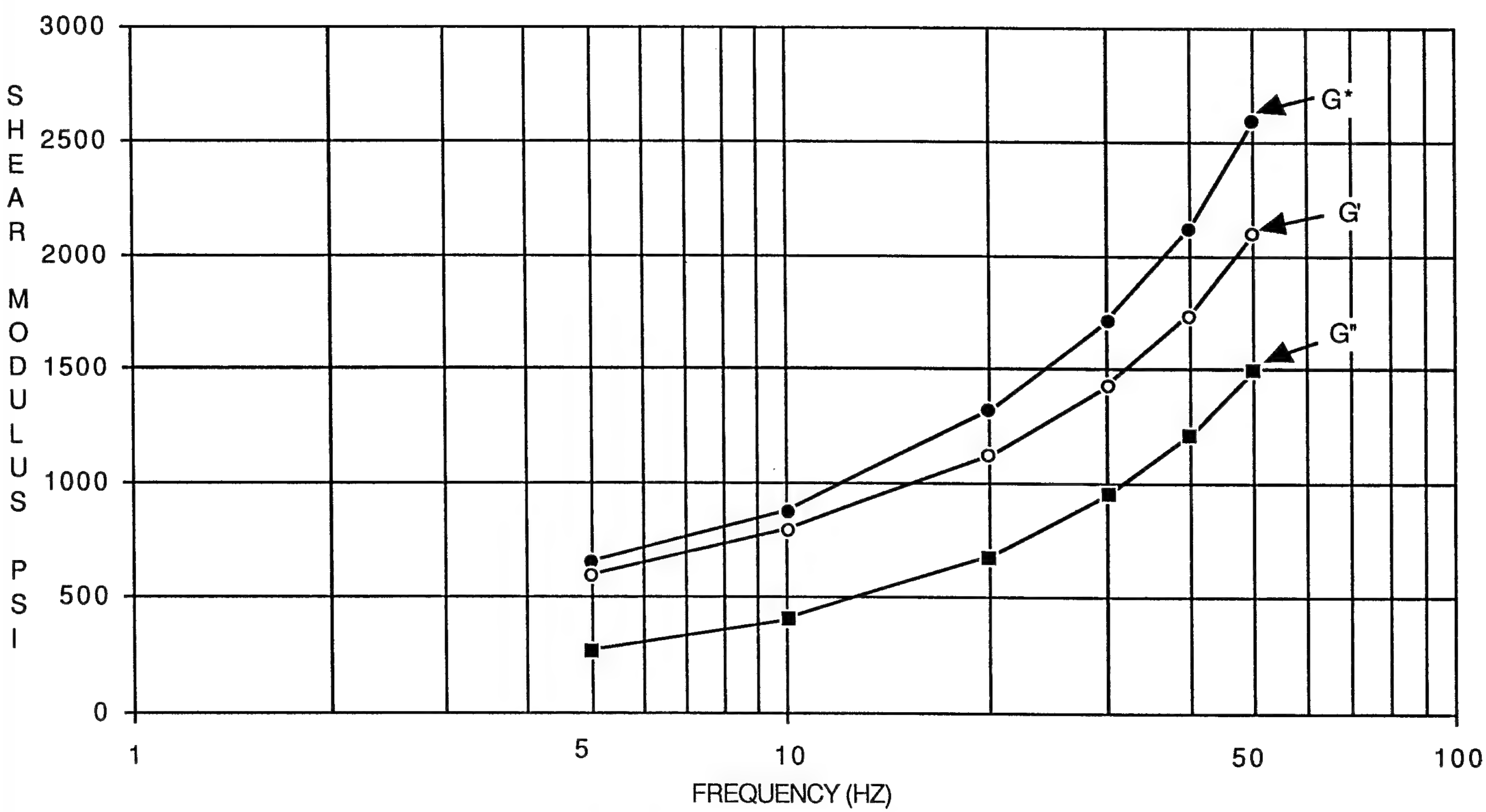

Figure 16. Dynamic shear modulus of TP-H1148 SRM propellant, mix 9910060 at 70°F, sample 36 from TWR-11779.

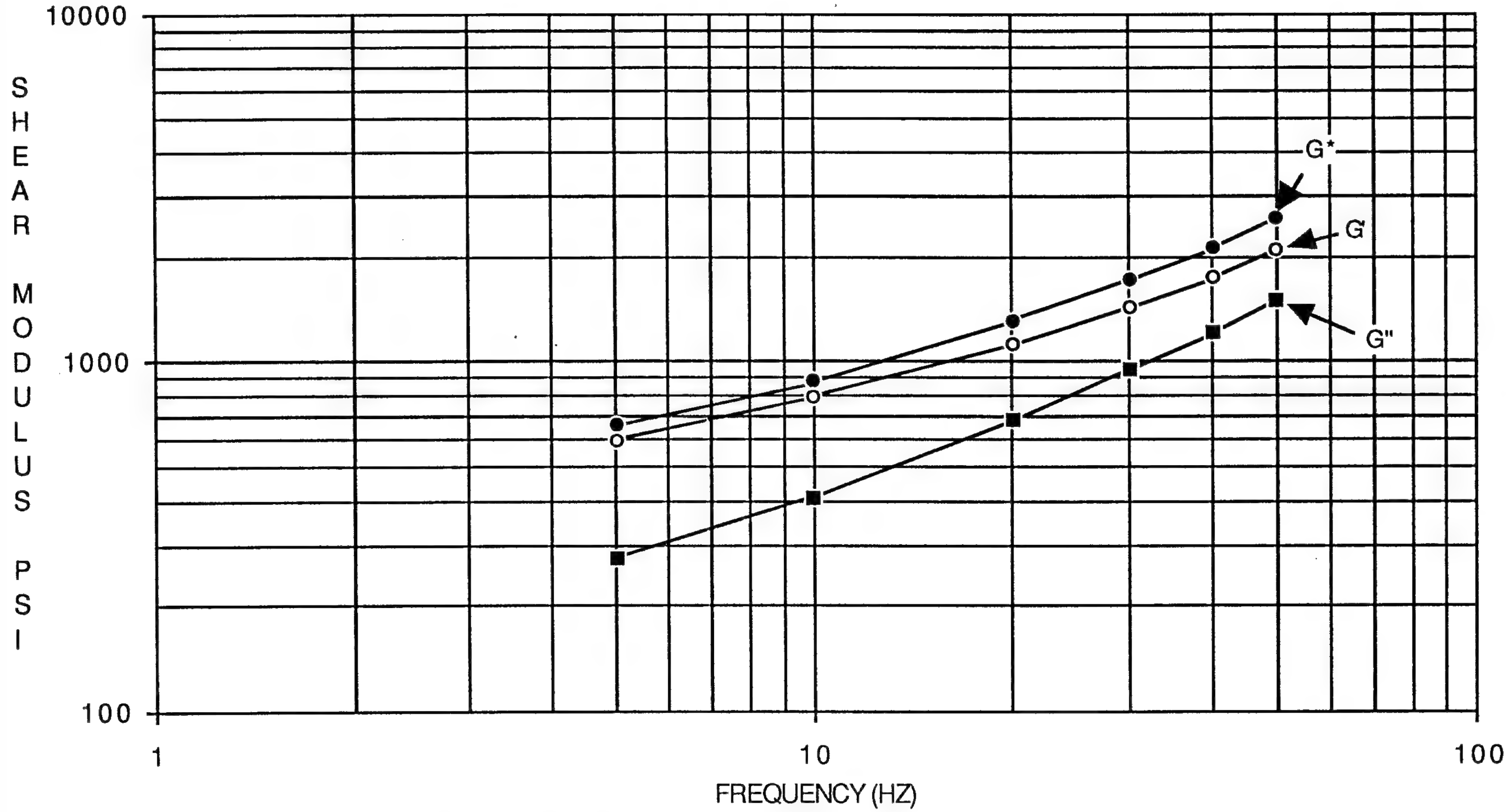

Figure 17. Dynamic shear modulus of TP-H1148 SRM propellant, mix 9910060 at 70°F, sample 36 from TWR-11779.

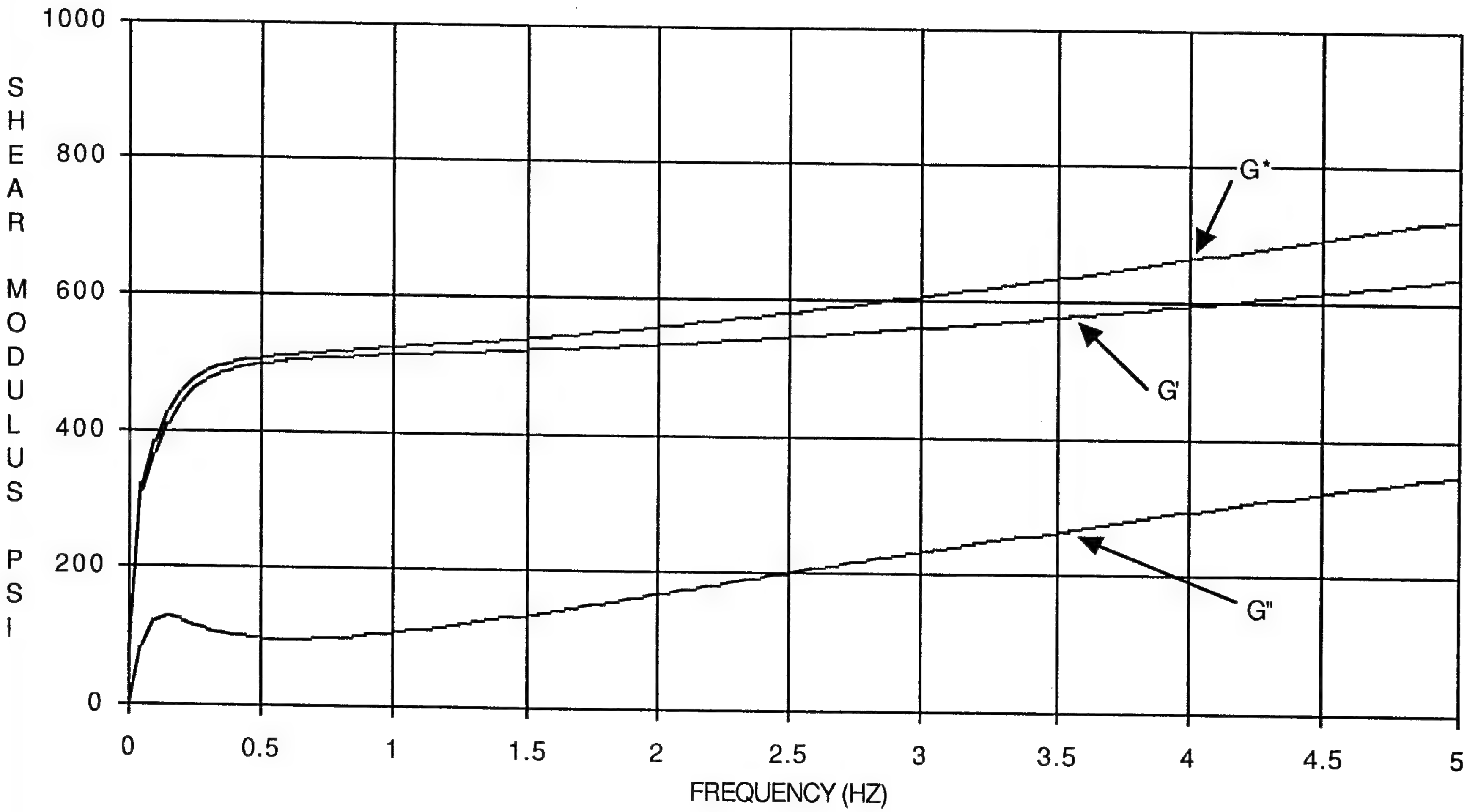

Figure 18. Dynamic shear modulus of TP-H1148 SRM propellant, based on Prony Series fit of stress relaxation data.

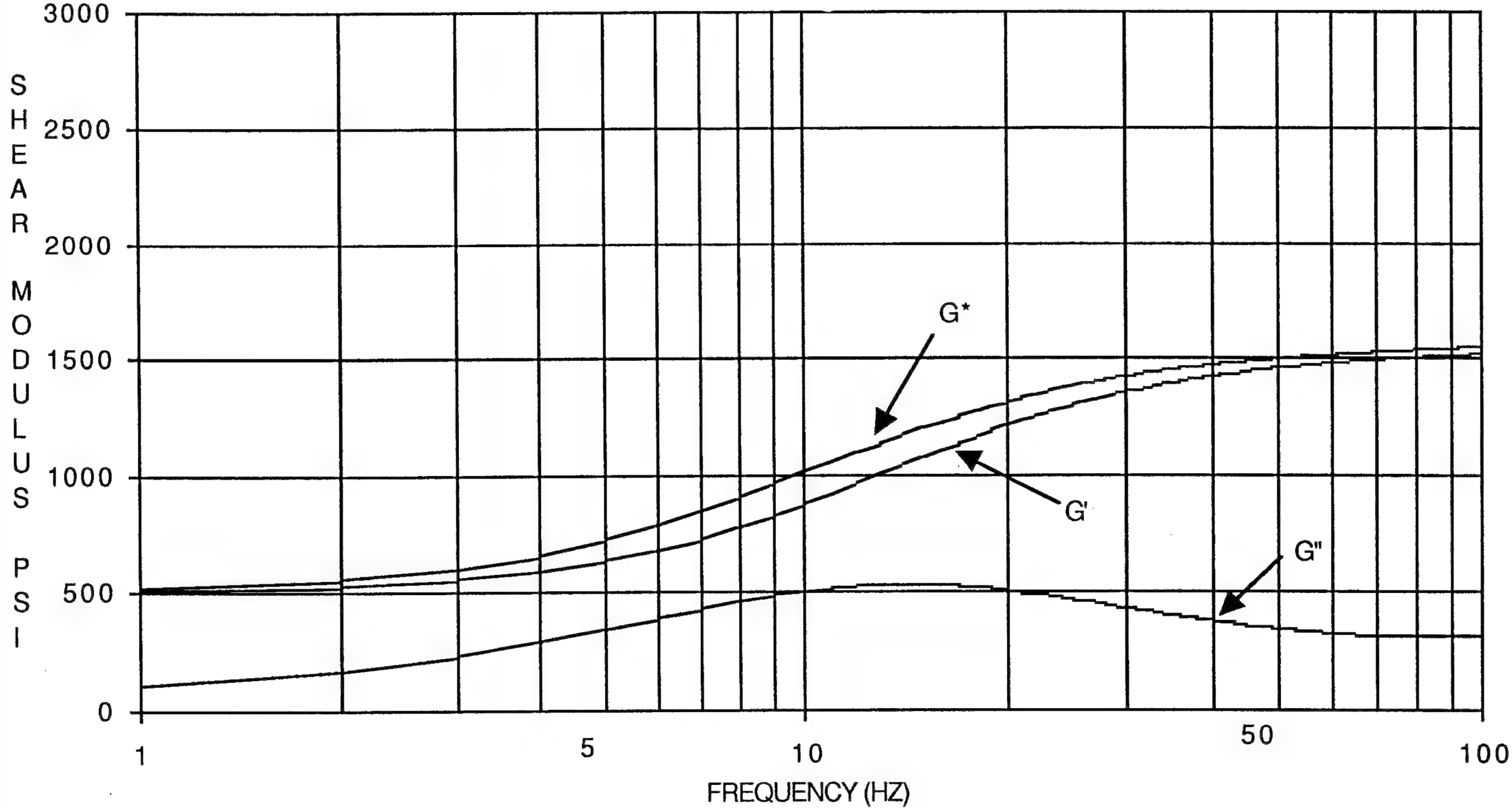

Figure 19. Dynamic shear modulus of TP-H1148 SRM propellant, based on Prony Series fit of stress relaxation data.

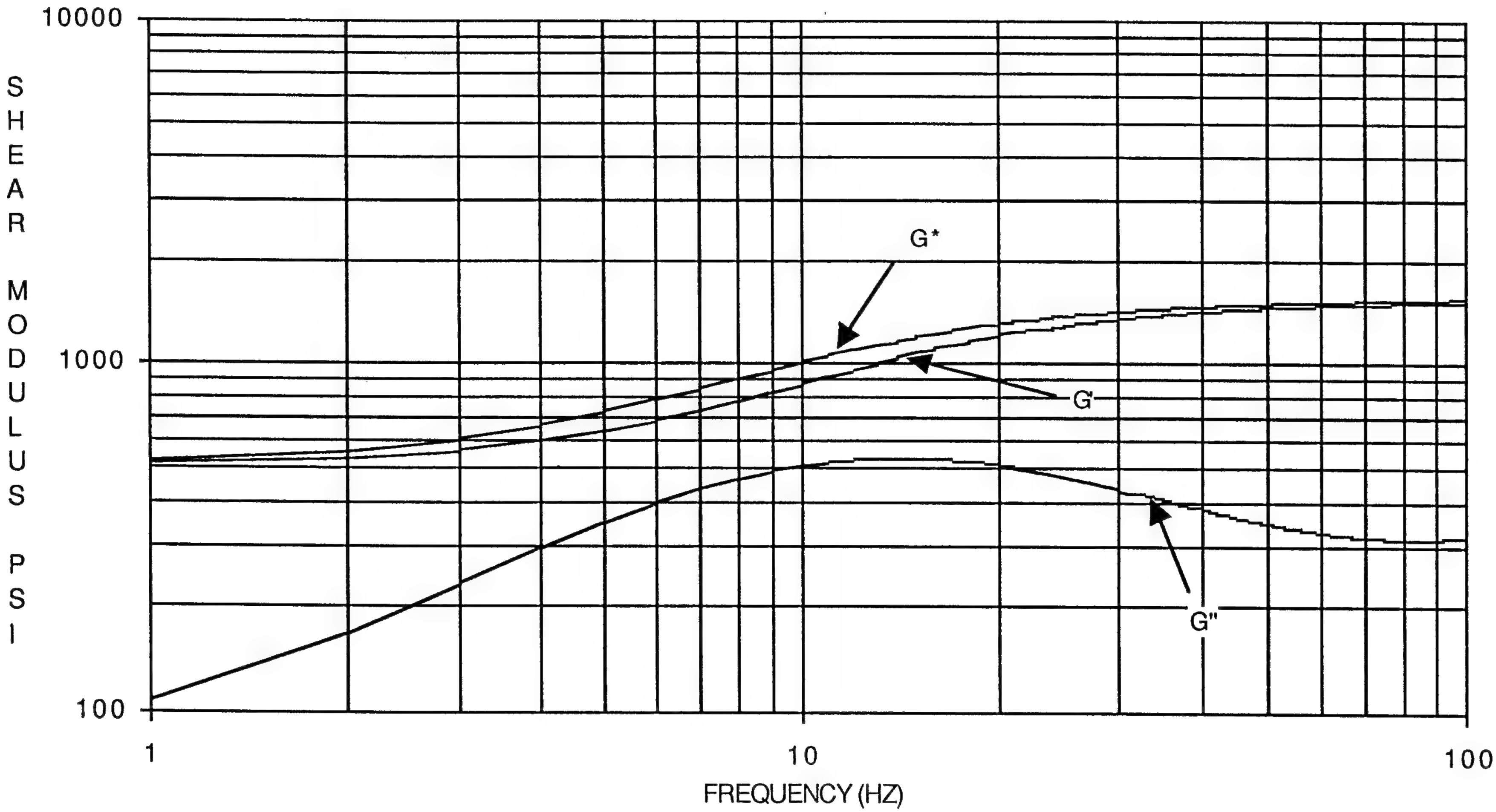

Figure 20. Dynamic shear modulus of TP-H1148 SRM propellant, based on Prony Series fit of stress relaxation data.

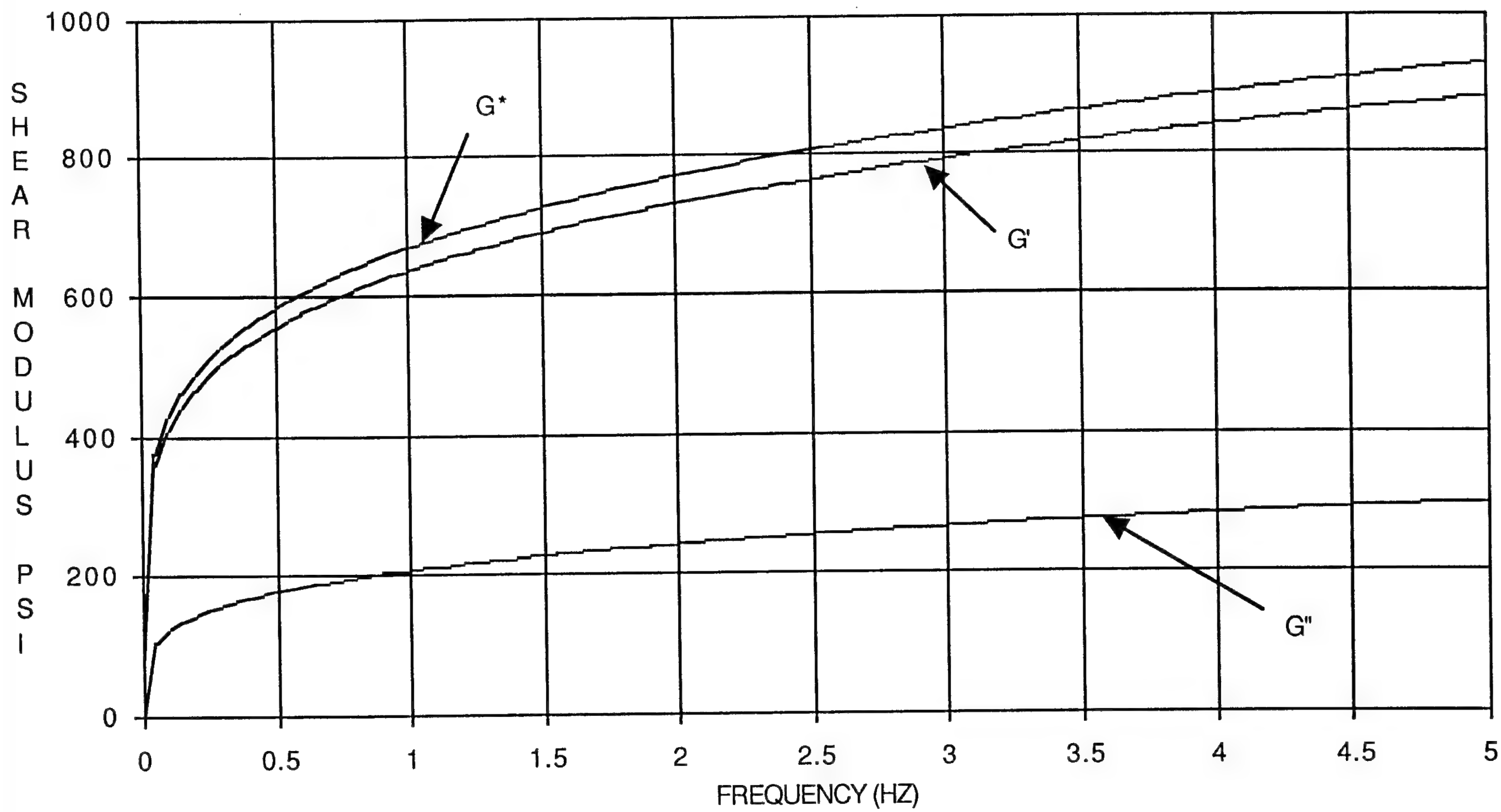

Figure 21. Dynamic shear modulus of TP-H1148 SRM propellant based on power equation fit of stress relaxation data.

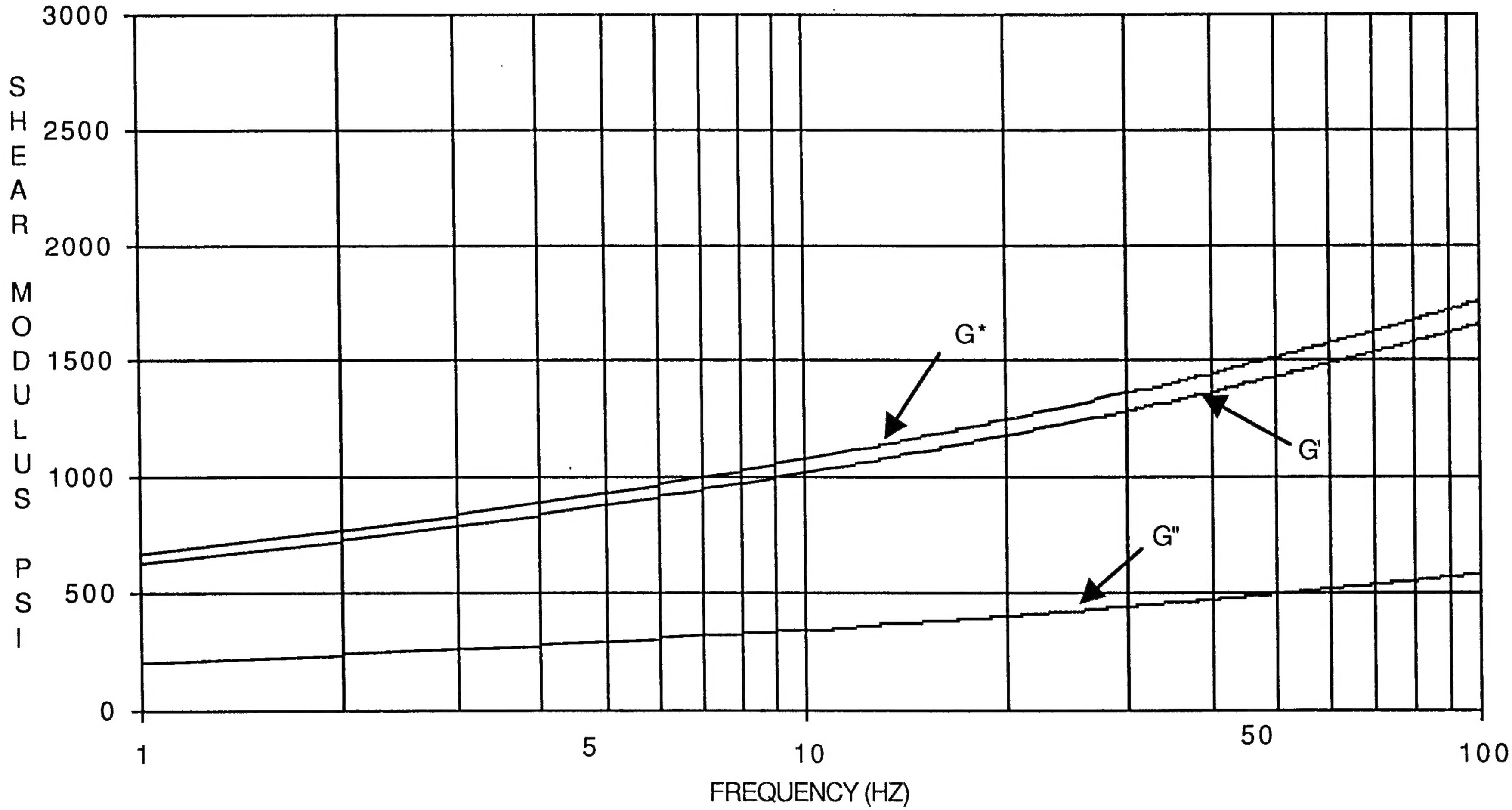

Figure 22. Dynamic shear modulus of TP-H1148 SRM propellant based on power equation fit of stress relaxation data.

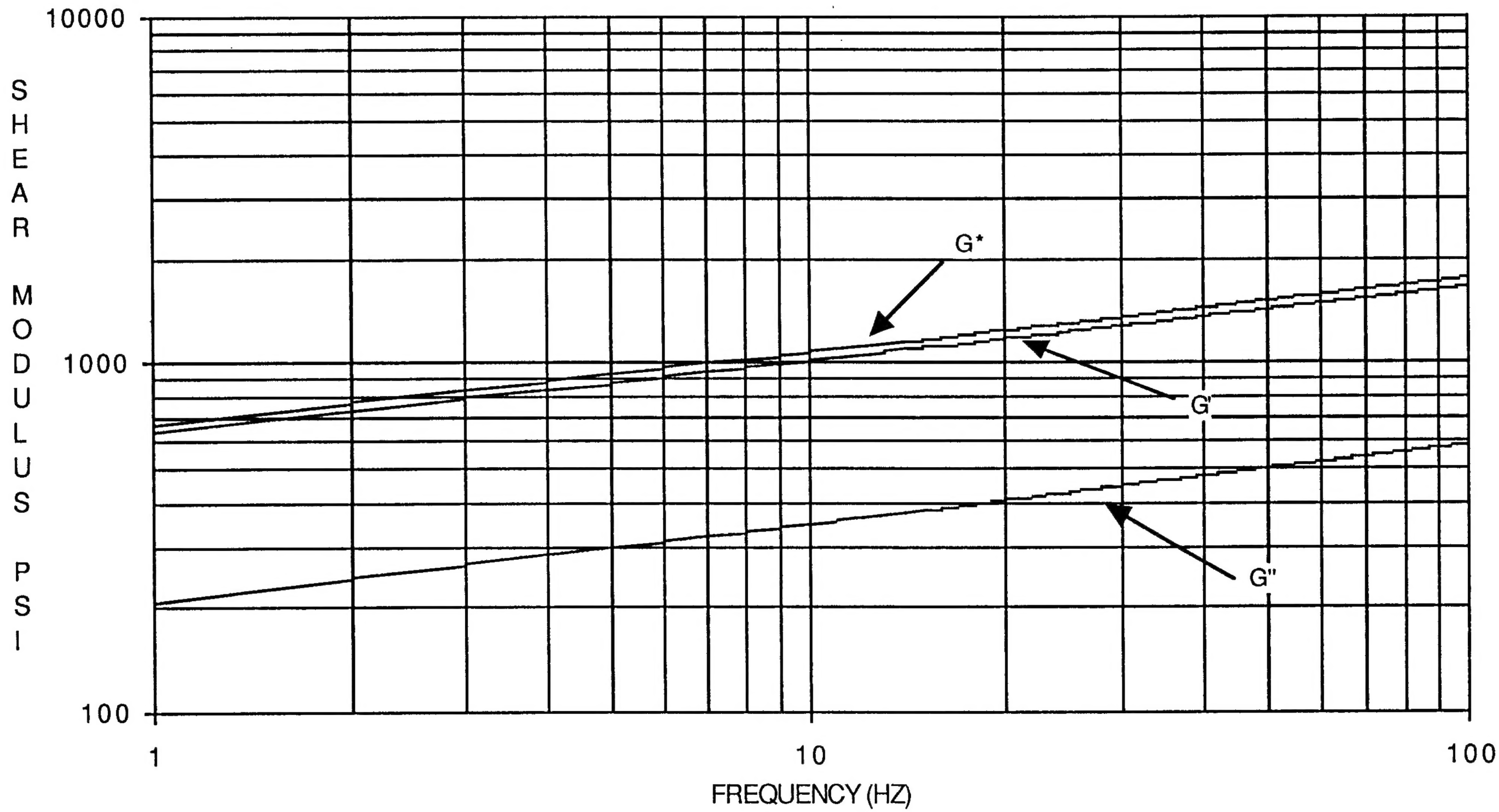

Figure 23. Dynamic shear modulus of TP-H1148 SRM propellant based on power equation fit of stress relaxation data.

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16. ABSTRACT

The following investigation reviews and evaluates the use of stress relaxation test data for the structural analysis of Solid Rocket Motor (SRM) propellants and other polymer materials used for liners, insulators, inhibitors, and seals. The stress relaxation data is examined and a new mathematical structural model is proposed for modeling. This model has potentially wide application to structural analysis of polymer materials and other materials generally characterized as being made of viscoelastic materials. A dynamic modulus is derived from the new model for stress relaxation modulus and is compared to the old viscoelastic model and experimental data.

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